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Bayesian Comparison Given a Type Two Censored Samples from a One Parameter Exponential Distribution

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Abstract. Bayesian comparison of the exponential distribution, based on type two censored life test, has been developed. For the five different prior distributions of the scale parameter θ, point and two sided Bayes interval estimates of θ have been obtained. Comparisons between these estimators with the non-Bayesian estimator, in the sense of Bayesian risk, have been carried out by using the exact and asymptotic Bayesian results. Finally, examination of the present results has been carried out using numerical example and computer facilities.

Introduction

The one parameter exponential distribution

$$f(t/\theta) = \frac{1}{\theta} \exp(-t/\theta) \qquad [t, \theta > 0]$$

where the scale parameter θ is itself a random variable distributed as

(I)
$$g(\theta) = \frac{\lambda^{g-1}}{\Gamma(g-1)} \theta^{-g} \exp(-\lambda/\theta)$$

$$[\mathrm{Re}(g) > 1, \lambda, \theta > 0]$$

be the inverted gamma density [IG (λ, g)] of [1, pp. 43-54, 227-229].

(II)
$$g(\theta) = \frac{(g-1)(\alpha\beta)^{g-1}}{\beta^{g-1} - \alpha^{g-1}} \theta^{-g}$$

$$[0 < \alpha \leq \theta \leq \beta < \infty, \mathrm{Re}(g) > 1]$$

be the uniform density [U(α,β,g) of [2, p. 49] on finite range (α,β).

(III) $$g(\theta) = \theta^{-g} \qquad [\theta > 0, \, Re(g) > 0]$$

be the prior quasi-density [Q(g)] of [2, p. 58] and

(IV) $$g(\theta) = (1/\lambda) \exp(-\theta/\lambda) \qquad [\lambda, \theta > 0]$$

be the exponential density [Exp (λ)] of [2, p. 49].

By suitably choice of the parameters (λ,g), one may be able to reasonably express the decision makers, prior judgement in terms of these prior densities. The parameters involved in (I), (II) and (III) give rise to an gamma and incomplete gamma functions with real arguements and for the parameters involved in (IV) gives rise to a Bessel function of type three.

Since we have a life test of type two censored sample, the likelihood function of the first r ordered failures, are observed, out of sample of size n is

$$L(t_{(1)}, t_{(2)}, \ldots, t_{(r)}/\theta) = \frac{n!}{(n-r)!}\, \theta^{-r} \exp.(-T/\theta)$$

where

$$T = \sum_{i=1}^{r} t_{(i)} + (n-r)\, t_{(r)}$$

The MVU estimator of θ is obtained by [3, pp. 468-502] as

$$\hat{\theta} = T/r \qquad (r > 0)$$

with

$$\mathrm{Var}(\hat{\theta}) = \theta^2/r$$

and ($2r\hat{\theta}/\theta$) distributed as a chi-square distribution with (2r) degrees of freedom.

Bhattacharya [2, pp. 50-59] developed Bayes estimator of θ for the prior densities (I), (II) and (IV), and Martz and Waller [4, pp. 342-378] discussed a Bayes point and interval estimator of θ for different prior densities including IG (λ, g + 1). The object of the present work is to make a comparison between these prior densities in the sense of relative Bayes risk. In fact, the best estimator of θ is that estimator with smallest Bayes risk. It has been noted that the two sided Bayes interval of θ for all prior densities presented here are given in terms of chi-square percentiles.

Prior Densities Over the Positive Real Line

Under the assumption of prior density (I)

We have

(1) The posterior density function, $f(\theta/T)$ is

$$f(\theta/T) = \frac{(\lambda + T)^{r+g-1}}{\Gamma\,(r+g-1)}\,\theta^{-(r+g)}\ \exp\{-(\lambda + T)/\theta\}, \qquad [\theta > 0,\ Re(r+g) > 1]$$

where

$$\Gamma\,(m + z) = (m - 1 + z)\,(m - 2 + z) \,.....\, (z + 1)\,(z)\,\Gamma\,(z)$$

$$[0 < Re(z) < 1 \text{ and } m = 1,2, \ldots]$$

(2) The S-th noncentral moment of θ is

$$E\,(\theta^S / T) = (\lambda + T)^S\,\Gamma\,(r+g-S-1)/\Gamma\,(r+g-1), \qquad S = 1,2, \ldots.$$

Especially, for $S = 1$ and 2, the Bayes estimator, θ_1^*, of θ and its variance are

$$\theta_1^* = E\,(\theta / T) = \frac{\lambda + T}{r + g - 2}, \qquad [Re(r+g)>2]$$

and

$$Var\,(\theta_1^*) = Var\,(\theta/T) = \frac{(\lambda + T)^2}{(r+g-2)^2\,(r+g-3)}, [Re(r+g)>3]$$

(3) The Bayes mode of θ, θ_2^*, is given by solving the equation $d\ln f(\theta/T)/d\theta = 0$, to yield

$$\theta_2^* = \frac{\lambda + T}{r + g}, \qquad [Re(r+g)> 0]$$

The pdf of θ_2^* is obtained as

$$f(\theta_2^*) = \Gamma^{-1}(r)\ \left(\frac{r + g}{\theta}\right)^r (\theta_2^* - \mu)^{r-1}\ \exp\left[-\,(r + g)\,(\theta_2^* - \mu)/\theta\right]$$

where

$$\theta_2^* > \mu\ ,\ \ \mu = \frac{\lambda}{r + g} > 0$$

The mean and the mean square error (MSE) of θ_2^* are

$$E(\theta_2^*) = \frac{r\,\theta + \lambda}{r + g}$$

and

$$MSE(\theta_2^*) = \frac{r\,\theta^2 + (\theta g - \lambda)^2}{(r + g)^2}$$

(4) A symmetric 100 (1 − p)% two sided Bayes probability interval (BI) estimate of θ is easily obtained by solving the two equations

$$\int_0^{\theta_0} f(\theta / T)\, d\theta = p/2$$

and

$$\int_{\theta_1}^{\infty} f(\theta / T)\, d\theta = p/2$$

for the lower limit θ_0 and the upper limit θ_1, so that

$$P_r \{\theta_0 < \theta < \theta_1\} = 1 - p$$

The interval (θ_0, θ_1) is referred to 100 (1 − p)% Bayes interval of θ. Solution of the above two equations yields

$$\theta_0 = 2\,(r + g)\,\theta_2^* / \chi^{2(1-p/2)}_{2(r+g-1)}$$

and

$$\theta_1 = 2\,(r + g)\,\theta_2^* / \chi^{2(p/2)}_{2(r+g-1)}$$

where $\chi_m^{2(a)}$ represents the 100 (a)-th percentile of a chi-square distribution with m degrees of freedom. Furthermore, a 100 (1-p)% lower and upper one sided Bayes interval estimate of θ may be obtained as

$$\theta_0 = 2\,(r + g)\,\theta_2^* / \chi^{2(1-p)}_{2(r+g-1)}$$

and

$$\theta_1 = 2\,(r + g)\,\theta_2^* / \chi^{2(p)}_{2(r+g-1)}$$

(5) The Bayes risk of $\hat{\theta}$, θ_1^* and θ_2^* are defined as

$$R(\hat{\theta}) = {}_0\int^{\infty} MSE(\hat{\theta})\, g(\theta)\, d\theta$$

$$= \frac{\lambda^2}{r(g-2)(g-3)}, \qquad [Re(g) > 3, r > 0]$$

$$R(\theta_1^*) = {}_0\int^{\infty} Var(\theta_1^*)\, f(T)\, dT$$

$$= \frac{\lambda^2}{(g-2)(g-3)(r+g-2)}, \qquad [Re(r+g) > 3]$$

where

$$f(T) = {}_0\int^{\infty} f(T/\theta)\, g(\theta)\, d\theta$$

$$= \frac{\lambda^{g-1}}{\beta(r,g-1)} T^{r-1} (\lambda+T)^{-(r+g-1)}, \qquad (T > 0)$$

and $\beta(m,z)$ be the beta function with integer m and real z. It has been noted that t/λ distributed as Beta of second type with parameter r and g-1.

Finally

$$R(\theta_2^*) = {}_0\int^{\infty} MSE(\theta_2^*)\, g(\theta)\, d\theta$$

$$= \frac{\lambda^2 (r+g+6)}{(r+g)^2 (g-2)(g-3)}$$

Now, the Bayes risk of θ_1^* and θ_2^* relative to the Bayes risk of $\hat{\theta}$ are

$$R(\theta_1^*, \hat{\theta}) = R(\theta_1^*) / R(\hat{\theta}) = r / (r+g-2)$$

and

$$R(\theta_2^*, \hat{\theta}) = R(\theta_2^*) / R(\hat{\theta}) = r(r+g+6) / (r+g)^2$$

It has been noted that, $R(\theta_1^*, \hat{\theta}) < R(\theta_2^*, \hat{\theta})$ and for this type of prior density that

$$R(\theta_1^*) < R(\hat{\theta}) < R(\theta_2^*)$$

In fact, the best estimator of θ in this case is θ_1^*

Under the assumption of prior density (IV)

(1) $$f(\theta/T) = K_{r-1}^{-1}\ (2\sqrt{T/\lambda})\,\theta^{-r} \exp\{-(\theta + T\lambda/\theta)/\theta\} \qquad (\theta > 0)$$

where $K_m(z)$ be the Bessel function of type three defined in [5, pp. 958-971]. For large values of λ, $f(\theta/T)$ will be equal $f(T = r\hat{\theta}/\theta)$.

(2) $$E(\theta^S/T) = (\lambda T)^{S/2}\ K_{r-m-1}\ (2\sqrt{T/\lambda})/K_{r-1}\ (2\sqrt{T/\lambda})$$

$$\simeq T^S\,(r-s-2)\,!\,/\,(r-2)\,!\,, \qquad (r > S+1,\ S = 1, 2, 3, \ldots.)$$

For S = 1 and 2, the Bayes estimator of θ and its variance are

$$\theta_1^* = \begin{cases} T\,K_{r-2}\ (2\sqrt{T/\lambda})/K_{r-1}\ (2\sqrt{T/\lambda}) \\ T/(r-2) \text{ for large } \lambda \text{ and } r > 2 \end{cases}$$

and

$$\mathrm{Var}(\theta_1^*) = \begin{cases} \lambda T\left[\dfrac{K_{r-3}\ (2\sqrt{T/\lambda})}{K_{r-1}\ (2\sqrt{T/\lambda})} - \left[\dfrac{K_{r-2}\ (2\sqrt{T/\lambda})}{K_{r-1}\ (2\sqrt{T/\lambda})}\right]^2\right] \\ \dfrac{T^2}{(r-2)\quad(r-3)} \text{ for large } \lambda \text{ and } r < 3 \end{cases}$$

Calculation of $K_{m+1}(z)$ for m = 0,1,2 are given in [6, pp. 359-436] and for other values of m, we have

$$K_{m+1}(z) = \frac{2m}{z}\,K_m(z) + K_{m-1}(z), \qquad m = 1,2,3, \ldots.$$

will be used to find other values of $K_m(z)$.

(3) The Bayes mode, θ_2^* is

$$\theta_2^* = \frac{-b + (b^2 + 4\lambda T)^{1/2}}{2} > 0$$

with the following pdf

$$f(\theta_2^*) = \frac{2(\lambda\theta)^{-r}}{\Gamma(r)} \left(\theta_2^* + \frac{\sqrt{b}}{2}\right) \left(\theta_2^{*2} + \theta_2^* \sqrt{b}\right)^{r-1}$$

$$\exp\{-(\theta_2^{*2} + \theta_2^* \sqrt{b}) / \lambda\theta\}, \qquad (\theta_2^* > 0)$$

where $b = r\lambda$ and for large λ, $f(\theta_2^*) = f(\hat{\theta})$

The mean and the variance of θ_2^* are defined as

$$E\left(\theta_2^* + \frac{\sqrt{b}}{2}\right) = \frac{e^G \sqrt{\lambda\theta}}{\Gamma(r)} \sum_{i=0}^{r-1} \binom{r-1}{i} (-G)^{r-i-1} \gamma\left(i + \frac{3}{2}, G\right)$$

and

$$\mathrm{Var}(\theta_2^*) = \frac{b}{4} + \lambda\theta - E^2\left(\theta_2^* + \frac{\sqrt{b}}{2}\right)$$

where

$$G = r/4\theta$$

and

$$\gamma(A,G) = \int_G^\infty X^{A-1} e^{-x} \, dx, \qquad [\mathrm{Re}(A) > 1, \mathrm{Re}(G) > 0]$$

The mean square error of θ_2^* can be easily obtained as

$$\mathrm{MSE}(\theta_2^*) = \mathrm{Var}(\theta_2^*) + [\theta - E(\theta_2^*)]^2$$

For large λ, we have

$$E(\theta_2^*) \simeq \theta$$

and

$$\mathrm{Var}(\theta_2^*) \simeq 2\lambda^2/r$$

(4) A 100(1-p)% two sided Bayes interval estimate of θ for large λ are

$$\theta_0 = 2T / \chi^{2(1-p/2)}_{2(r-1)}$$

and

$$\theta_1 = 2T / \chi^{2(p/2)}_{2(r-1)}$$

Also, a one sided Bayes interval can be easily obtained from θ_0 for the lower limit and from θ_1 for the upper limit by replacing p/2 with p.

(5) The Bayes risk of $\hat{\theta}$, θ_1^* and θ_2^* are

$$R(\hat{\theta}) = 2\lambda^2 / r, \qquad (\eta > 0)$$

$$R(\theta_1^*) = 2\lambda^2 / (r+2)$$

where

$$f(T) = \frac{2}{\lambda \Gamma(r)} \left(\frac{T}{\lambda}\right)^{(r-1)/2} K_{(r-1)} \left(2\sqrt{T/\lambda}\right)$$

and

$$R(\theta_2^*) \simeq 2\lambda^2/r \qquad \text{for large } \lambda \text{ and } r > 0$$

Now, the relative Bayes risk of θ_1^* and θ_2^* are

$$R(\theta_1^*, \hat{\theta}) = r/(r+2)$$

and

$$R(\theta_2^*, \hat{\theta}) = 1.0$$

We note that $R(\theta_1^*) < R(\hat{\theta}) < R(\theta_2^*)$ and the best estimator of θ in this case is θ_1^*.

Under the assumption of prior density (III)

(1)
$$f(\theta/T) = \frac{T^{r+g-1}}{\Gamma(r+g-1)} \theta^{-(r+g)} e^{-T/\theta}, \qquad [Re(r+g) > 1, \theta > 0]$$

(2) The S-th non-central moment of θ

$$E(\theta^S/T) = T^S \Gamma(r+g-1) / \Gamma(r+g-1)$$

Especially, for $S = 1$ and 2, we have

$$\theta_1^* = \frac{T}{r+g-2}, \qquad [Re(r+g)>2]$$

and

$$Var(\theta_1^*) = \frac{T^2}{(r+g-2)^2 (r+g-3)}, \qquad [Re(r+g)>3]$$

(3) The Bayes mode, θ_2^* of θ is

$$\theta_2^* = T/(r+g)$$

and $[2(r + g)\theta_2^*]/\theta$ distributed as a chi-square distribution with (2r) degrees of freedom.

The mean and the mean square error of θ_2^* are

$$E(\theta_2^*) = \frac{r\ \theta}{r+g}$$

and

$$MSE\ (\theta_2^*) = \frac{\theta^2\ (r + g^2)}{(r + g)^2}$$

(4) A 100 (1-p)% two sided Bayes interval are

$$\theta_0 = 2\ (r + g)\theta_2^* \,/\, \chi^{2(1-p/2)}_{2(r+g-1)}$$

and

$$\theta_1 = 2\ (r + g)\theta_2^* \,/\, \chi^{2(p/2)}_{2(r+g-1)}$$

A lower and upper one sided Bayes interval of θ can.be obtained easily as before with p/2 replaced with p in θ_0 and θ_1

(5) Without loss of generality, we assume that $\theta > \alpha_0$, $\alpha_0 > 0$..

Then, the Bayes risk of $\hat{\theta}$, θ_1^* and θ_2^* are obtained as

$$R(\hat{\theta}) = (\frac{g - 1}{g - 3})\ \frac{\alpha_0^2}{r}\ , \qquad [Re(g)>3]$$

$$R(\theta_1^*) = (\frac{g - 1}{g - 3})\ \frac{\alpha_0^2}{(r+g-2)^2\ (r+g-3)}\ , \qquad [Re(r+g)>3]$$

where

$$f(T) = (g - 1)\ \alpha_0^{g-1}\ T^{-g}, \qquad [T > \alpha_0 > 0,\ Re(g) > 1]$$

and

$$R(\theta_2^*) = (\frac{g - 1}{g - 3})\ \frac{\alpha_0^2\ (r + g^2)}{(r+g)^2}$$

Now, the relative Bayes risk of θ_1^* and θ_2^* are

$$R(\theta_1^*, \hat{\theta}) = r/(r+g-2)^2\,(r+g-3),$$

and

$$R(\theta_2^*, \hat{\theta}) = r/(r+g^2)\,/\,(r+g)^2$$

we note that $R(\theta_1^*) < R(\theta_2^*) < R(\hat{\theta})$ and θ_1^* is the best estimator for θ in this case.

If g = 0 above yields the corresonding results for the non-informative prior that is uniform over $[0, \alpha]$, and if g = 2, θ_1^* reduces to $\hat{\theta}$. Bhattacharya [2, pp. 60-61] proved that, $g(\theta) = \theta^{-2}$ is the only prior which leads to the MVU estimator of θ.

Prior Density over the Positive Finite Range

In this section we shall consider the case of prior density (I)

(1) $$f(\theta/T) = \begin{cases} \dfrac{T^{r+g-1}}{\Gamma^*(r+g-1,y)}\ \theta^{-(r+g)}\ e^{-T/\theta}, & (\alpha < \theta < \beta) \\[2ex] \dfrac{T^{r+g-1}}{\gamma(r+g-1,T/B)}\ \theta^{-(r+g)}\ e^{-T/\beta}, & (0 < \theta < \beta) \end{cases}$$

where

$$\Gamma^*(A,y) = \Gamma(A,T/\alpha) - \Gamma(A,T/\beta)$$

$$\Gamma^*(A,y) = \int_0^y x^{A-1}\ e^{-x}\ dx\,, \qquad [Re(A) > 1, y > 0]$$

and

$$\gamma(A,y) = 1 - \Gamma(A,y)$$

(2) The S-th non central moments of θ is

$$E(\theta^S/T) = \begin{cases} T^S\left\{\dfrac{\Gamma^*(r+g-S-1,y)}{\Gamma^*(r+g-1,y)}\right\} & (\alpha < \theta < \beta) \\[3ex] T^S\left\{\dfrac{\gamma(r+g-S-1,T/\beta)}{\gamma(r+g-1,T/\beta)}\right\} & (\alpha < \theta < \beta) \end{cases}$$

For S = 1 and 2, we have

$$\theta_1^* = \begin{cases} T\left\{ \dfrac{\Gamma^*(r+g-2,y)}{\Gamma^{*2}(r+g-1,y)} \right\} & (\alpha < \theta < \beta) \\ \\ T\left\{ \dfrac{\gamma(r+g-2, T/\beta)}{\gamma(r+g-1, T/\beta)} \right\} & (\alpha < \theta < \beta) \end{cases}$$

and

$$\mathrm{Var}(\theta_1^*) = \begin{cases} T^2\left\{ \dfrac{\Gamma^*(r+g-3,y)\,\Gamma^*(r+g-1,y) - \Gamma^{*2}(r+g-2,y)}{\Gamma^{*2}(r+g-1,y)} \right\} & \text{as } \alpha < \theta < \beta \\ \\ T^2\left\{ \dfrac{\gamma(r+g-3, T/\beta)\,\gamma(r+g-3, T/\beta)\,\gamma^2(r+g-2, T/\beta)}{\gamma^2(r+g-1, T/\beta)} \right\} & \text{as } \alpha < \theta < \beta \end{cases}$$

Incomplete gamma subroutines can be used to calculate θ_1^* and Var (θ_1^*).

(3). The Bayes mode of θ is given as for $\alpha < \theta < \beta$

$$\theta_2^* = \begin{cases} \alpha & \text{if } T/(g+r) < \alpha \\ T/(g+r) & \text{if } \alpha < T/(g+r) < \beta \\ \beta & \text{if } T/(g+r) > \beta \end{cases}$$

or as $0 < \theta < \beta$

$$\theta_2^* = \begin{cases} T/(g+r) & \text{if } 0 < T/(g+r) < \beta \\ \\ \beta & \text{if } T/(g+r) > \beta \end{cases}$$

Since $\alpha > 0$ and $\beta < \infty$, then without loss of generality we can assume $0 < \theta_2^* < \infty$ and the quantity $2\,(r+g)\,\theta_2^*/\theta$ distributed as a chi-square distribution with (2r) degrees of freedom.

The mean and the mean square error of θ are given as

$$E(\theta_2^*) = \frac{r\theta}{r + g}$$

and [Re (r + g) > 0]

$$MSE\ (\theta_2^*) = \frac{\theta^2 (r + g^2)}{(r + g)^2}$$

(4). A 100 (1-P)% two sided Bayes interval of θ is obtained by solving the following two equations

$$\frac{P}{2} = \frac{\Gamma (r + g - 1, T/\alpha) - \Gamma (r + g - 1, T/\theta_0)}{\Gamma^* (r + g - 1, y)}$$

and

$$\frac{P}{2} = \frac{\Gamma (r + g - 1, T/\theta_1) - \Gamma (r + g - 1, T/\beta)}{\Gamma^* (r + g - 1, y)}$$

in θ_0 and θ_1 or in terms of chi-square percentilies

$$\theta_0 = 2T/\chi^{2(v_1)}_{2(r+g-1)}$$

and

$$\theta_1 = 2T/\chi^{2(v_2)}_{2(r+g-1)}$$

where

$$v_1 = \frac{\Gamma (r + g - 1, T/\alpha) (2 - P) + P\,\Gamma (r + g - 1, T/\beta)}{2\,\Gamma (r + g - 1)}$$

and

$$v_2 = \frac{P\,\Gamma (r + g - 1, T/\alpha) + \Gamma (r + g - 1, T/\beta) (2 - P)}{2\,\Gamma (r + g - 1)}$$

Note that $0 < v_2 < b_1 < 1$ and $\alpha < \theta < \beta$

For $0 < \theta < \beta$, we have

$$\theta_0 = 2T / \chi^{2(1-v)}_{2(r+g-1)}$$

and

$$\theta_1 = 2T / \chi^{2(v)}_{2(r+g-1)}$$

where

$$0 < v = \frac{P\,\gamma\,(r+g-1, T/\beta)}{2\,\gamma\,(r+g-1)} < \frac{P}{2}$$

Furthermore, a 100 (1-P)% lower and upper one sided Bayes interval estimate of θ may be obtained easily by replacing v_1, v_2 and v by $2v_1$, $2v_2$ and 2v respectively in θ_0 and θ_1.

(5) The Bayes risk of $\hat{\theta}$, θ_1^* and θ_2^* are

$$R(\hat{\theta}) = \frac{(g-1)(\beta^{g-3} - \alpha^{g-3})(\alpha\beta)^2}{(g-3)(\beta^{g-1} - \alpha^{g-1})\,r}, \qquad [\mathrm{Re}\,(g) > 3]$$

$$R(\theta_1^*) = \frac{(g-1)(\beta^{g-3} - \alpha^{g-3})(\alpha\beta)^2}{(g-3)(\beta^{g-1} - \alpha^{g-1})(r+g-2)}$$

where

$$f(T) = \frac{(g-1)(\alpha\,\beta)^{g-1}\ T^{-g}}{\Gamma(r)(\beta^{g-1} - \alpha^{g-1})}\Gamma^*(r+g-1, y), \qquad [T > 0]$$

$$R(\theta_2^*) = \frac{(g-1)(\beta^{g-3} - \alpha^{g-3})(\alpha\,\beta)^2 (r+g^2)}{(g-3)(\beta^{g-1} - \alpha^{g-1}(r+g)^2}$$

Now, the relative Bayes risk of θ_1^* and θ_2^* are

$$R(\theta_1^*, \hat{\theta}) = r/(r+g-2)^2,$$

and

$$R(\theta_2^*, \hat{\theta}) = r(r+g^2) / (r+g)^2,$$

We note that $R(\theta_1^*) < R(\theta_2^*) < R(\hat{\theta})$ and θ_1^* is the best estimator of θ in this case.

If g = 0, the corresponding results in the case of $U(\alpha,\beta)$ are obtained as a special case from the present results.

Numerical Example

Considering the example given in [4,p. 349] in which 5 failures were observed with total time on test T equal 1.6×10^5 h., the prior parameters chosen were $g = 8.5$ and $\lambda = 2.86 \times 10^5$ and α & β for the uniform prior density were taken as 2.0×10^4 and 7.0×10^4 such that $2.0 \times 10^4 < \theta < 7.0 \times 10^4$. Now, the maximum likelihood estimator of θ is 3.2×10^4h. with variance $= 0.2\,\theta^2$. The following table shows the estimate values of the resulting formulas in hours.

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Estimates of θ with relative risk for 5 prior distributions

Prior df.	Point Bayes estimator		95% two sided BI		95% one sided BI		Relative Bayes Risk	
	θ_1^*	θ_2^*	θ_0	θ_1	θ_0	θ_1	$R(\theta_1^*, \hat{\theta})$	$R(\theta_2^*, \theta)$
IG (λ,g)	38792.61 (1.43×10^8)	33037.04 (2.99×10^7)	21945.58	67941.20	23690.64	61049.89	0.43478	0.53498
E(λ)	49420 (9.15×10^8)	62628.53 (1.79×10^9)	18249.22	146788.9	20635.84	117087.5	0.83333	1.0
Q(g) $\alpha_0 = 1$	13913.04 ($1.84 - 10^7$)	11851.85 (3.85×10^8)	7872.850	24373.52	8498.880	21901.31	0.00360	2.11934
U (α,β)	23640.49 (1.44×10^7)	2.0×10^4 (1.09×10^7)	21813.22	30476.19	17322.57	30418.25	0.03781	2.11934
U(0,β)	13913.18 (1.84×10^7)	11851.85 (3.85×10^6)	7874.020	24390.80	8498.880	21901.31	0.03781	2.11934

Where numbers between brackets defines Var (θ_1^*) and Var (θ_2^*) evaluated at $\theta = \theta_1^*$ or θ_2^*.

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أستاذ مشارك، قسم الأساليب الكمية، كلية العلوم الإدارية،
جامعة الملك سعود، الرياض، المملكة العربية السعودية

ملخص البحث. البحث يتناول المقارنة البيزيه باستخدام اختبارات الحياة المراقبة من النوع الثاني. ففي هذا البحث استخدمنا خمسة أنواع من التوزيعات الاحتمالية القبلية لمعلمة الموضع وتم الحصول على تقديرات لمعلمة الموضع تحت فرض تلك التوزيعات الاحتمالية الخمس باستخدام نظرية التقدير بنقطة ونظرية التقدير بفترة وتمت المقارنة بين تلك المقدرات باستخدام دالة الخطر البيزيه النسبية. وفي النهاية تمت المقارنة واختيارُ أفضل التقديرات باستخدام بيانات فعلية تم تنفيذها على الحاسب الآلي.