

A Comment On:

A Stochastic Approach to the Estimation of the Numbers of Survivals at Each Age

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The aim of this study is to discuss the paper entitled “A stochastic approach to the estimation of the number of survivals at each age”[1] which treated the same problem as our previous paper “*An analytical adjustment of life tables*”[2] with alternative approach.

I–The Claim that the Phenomenon of Birth and Death are Random Processes

The criticized paper claimed that the phenomenon of birth and death are random processes and should be so treated. For this purpose it showed demographic data for most of the countries of the world, and it claimed that this was not clear in the previous studies for this phenomenon.

In fact the phenomenon of birth and death is very old and it was known from old times. Human beings have realized that this phenomenon is a random process and this can be found in the old Arabic literature.

When we used a mathematical expression for the survival function as Makeham’s law:

$$l_x = ks^x g^{c^x}$$

this is not to say that the random nature of the phenomenon has been rejected. We never claimed that the number of survivals at each age obeys exactly Makeham’s law. This is very clear in page 23 of Bakaga Gi book which is one of the criticized paper

references[3, p. 23] “this is not to say that every one must die according to this law, but the probability of his death according to this law is better of more frequent occurrence”.

Moreover the elementary books of insurance simplify the mortality tables by mentioning that the survivals and deads numbers at each age from a fictitious group of people can be used for determining the empirical probabilities so as to calculate the insurance premiums [4, p. 316]. It is well known that the empirical probabilities have not absolute values, but these are different from place to place and from time to time. Therefore, we must revise these mortality tables from time to time [5, p.85]. This is equivalent to saying that the number of survivals at any age is a random variable. Is there any need to demonstrate that the phenomenon of birth and death are random processes in a scientific paper?

II-The Estimation of the Number of Survivals at Each Age

Although the title of the criticized paper is “*A Stochastic approach to the estimation of the number of survivals at each age*”, we can’t find the estimated values of survivals numbers in the criticized paper to compare them with real values or other estimates.

III-The Simplicity and the Superiority

The criticized paper claimed that a simple method is given to estimate the number of the survivals at each age and this method is superior to other methods. However, this claim has not been supported by a scientific proof.

IV-Criticism to the Suggested Method

The suggested method is known and is used in different branches of science, but it is not used in this problem because:

- Correlations exist between the survivals numbrs at different ages.
- The accuracy of estimation is inadequate, because the criticized paper assumes that the estimated values of the parameter c increase with increments of 10^{-2} , and after this the suggested method finds the remaining parameters for each assumed value of the parameter c . Therefore, we always find the estimated value of the parameter c with an error at a range of 10^{-2} . This is unacceptable because the Makeham’s law has the power c^x and the values of x may be very large (100 or more) so the error increases very much. If we want to decrease this error, we must assume estimated values of the parameter c which increases with increments of 10^{-6} – the required accuracy of estimation in this case– Thus the suggested method will be very complex if not impossible to apply. Moreover the criticized paper did not prove that R^2 is a decreasing function for $c > 1.10$.

V-Comparing the Results

The criticized paper has used the same data that we have used, but it did not compare the estimated values of both papers. The following table gives the real values and the estimated values from the criticized paper of the survivals numbers at each age:

Age	Real value	Estimated value	Age	Real value	Estimated value
35	91552	91573	55	81640	81780
36	91334	91367	56	80548	80747
37	91104	91145	57	79378	79629
38	90860	90905	58	78127	78420
39	90600	90646	59	76789	77115
40	90322	90366	60	75355	75709
41	90024	90062	61	73822	74193
42	89705	89733	62	72190	72565
43	89363	89376	63	70453	70818
44	88994	88989	64	68602	68947
45	88592	88569	65	66625	66950
46	88152	88113	66	64514	64822
47	87607	87618	67	62270	62562
48	87141	87080	68	59891	60169
49	86560	86496	69	57376	57646
50	85920	85862	70	54725	54994
51	85215	85173	71	51947	52220
52	84438	84425	72	49058	49332
53	83585	83614	73	46070	46342
54	82653	82734	74	42994	43263

We have used the following parameter values for Makeham's law:

$$k = 94655.382; s = 0.999576; g=0.999350; c = 1.10$$

These are the estimated values which correspond to the maximum value of the coefficient of determination. The sum of absolute deviations between the estimated values and the real values is: 6584, it is greater than our one. Therefore, we conclude that our method gives better results.

If we want to compare the simplicity of both methods we have to look at the theoretical bases of the criticized method with respect to ours. The criticized paper did not make such comparison, but it has compared the theoretical basis of our method with its computer results.

References

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- [4] **Abdulla, S.** *Risk and Insurance*. Cairo: Dar Al-Nahda Al-Arabia, 1976, (in Arabic language).
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