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A Fuzzy Goal Programming Approach to Resource Allocation Problem: A Case Study

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Abstract. An additive fuzzy goal programming (FGP) model with pre-emptive priority structure is used for allocating scare resources among competing economic sectors in a developing economy. Only limited applications of FGP related to macro-economic development planning have been presented in the literature. In that regard, this paper describes briefly the unique problem of resource allocation in Saudi Arabia, the specific FGP model developed, and the application of the model using real data.

The FGP model is shown to be an adequate supporting aid for developing and evaluating alternative strategies for allocating economic resources.

Keywords: Fuzzy set theory; Multiple criteria; Fuzzy goal programming; Resource allocation problem; Planning.

Introduction

Goal Programming (GP) is one of the multi-objective techniques that is widely applied for modelling real world decision problems. It requires specifying aspiration levels for the objectives and aims at reducing the deviations from these aspiration levels. A major

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drawback of GP approach is that it requires the decision makers to specify both priorties and goal values a priori. Often, in real life problems the aspiration levels and/or priority factors of the decision maker, and sometimes even the weights to be assigned to the goals, are imprecise in nature. In such situations the use of fuzzy set theory (Zadeh [1]) allows vague aspirations of the decision maker to be quantified and used in a decision making context.

The use of fuzzy set theory in GP was first investigated by Narasimhan [2], Hannan [3,4,5], Narasimhan [6], Ignizio[7], Rubin and Narasimhan [8]. Tiwari *et al* [9,10] have investigated a particular additive model (weighted and preemptive) in fuzzy goal programming (FGP) context to aggregate the fuzzy goals.

This paper presents an application of a fuzzy goal programming approach (Tiwari *et al.* [10]) to multiple-objective resource allocation problem as a case study in Saudi Arabia.

Problem Description

Saudi Arabia vast financial resources have enabled it to undertake massive scale development efforts. These efforts have been planned and most often financed by the government to guide the process of change in a coordinated and balanced direction. Since 1970 the government has launched a sequence of Five-Year Development plans to accelerate the rate of economic and social progress. The basic objectives of these plans have been more rapid economic growth, diversifying sources of national income and reducing the country's dependence on oil, raising living standards and improving the quality of life, maintaining economic and social stability, and developing human resources [11].

During the seventies, the supply and demand conditions induced the value of oil export to grow very rapidly. The increase in oil revenues enabled the government, which is the sole recipient of these revenues, to increase its development expenditures. Because of the country's high dependence on oil as the major source of foreign exchange and the realization of policy makers of the consequences of any drastic shifts in the world's demand for oil, the government initiated massive scale efforts to diversify the sources of national income . In particular, this effort involved raising the investment/GDP ratio in the non-oil sectors of the economy with a view to implanting a permanent basis for a fully operational and economically viable industrial economy. The growth of aggregate investment proceeded at a rapid pace such that the structure of domestic supplies of commodities and labor was unable to match the resulting growth in demand. These rigidities, however, have been overcome by massive inflows of goods and laborers.

Although the diversification policy has emphasized investments in capital intensive industries, the economy continued to depend for further development on a large inflow of expatriate workforce. With a high level of expatriate labor force in the country, policy makers began to contemplate the economic implications of providing subsidized

housing, transportation, communication and electricity, food and energy subsidies , and free health and education services to a rapidly expanding population of foreigners. The need to keep the expatriate population down to a politically and culturally tolerable level became more evident as planners began to realize that the social costs of the foreign labor force may in some cases exceed their contribution to GDP, and hence undermine the benefits of diversification.

The development experience of Saudi Arabia has demonstrated a presence of several conflicting development objectives and continuous shifts in the priorities of these objectives. The problem of the growing expatriate workforce has acquired increased urgency because of social and political implications of their continuous presence. At the same time the achievements of growth and economic independence are also critical to the success of economic development. A realization of the multiple nature of development objectives and a consideration of the historical experience of the continuous shifts in the priorities of these objectives suggest that evaluation and allocation of national resources might be conducted on the basis of a multi-objective analytical framework.

Fuzzy Goal Programming Approach

The General GP Problem (Simple additive model) for m goals $G_i(\chi)$ with deviational variables is as follows :

$$\text{Minimize} \sum_{i=1}^{m} (d_i^+ + d_i^-)$$

Subject to:

$$\begin{aligned} & G_i(\chi) + d_i^- - d_i^+ = g_i, \\ & d_i^+ \cdot d_i^- = 0, \\ & d_i^+, d_i^- = 0, \\ & d_i^+, d_i^-, \chi \geq 0, i = 1,2,\ldots,m. \end{aligned} \qquad (1)$$

where g_i is the aspiration level of the i-th goal.

Model (1) can be fuzzified by the concept of FGP (see Tiwari *et al.* [10]) using membership function instead of deviational variables. A membership function yields a degree of closeness of each goal to its desired attainment level using a zero to one interval to represent the grade of membership of each goal. The worst possible value for an objective function has a grade of membership of zero. The membership function for each goal may be elicited by asking the decision maker about the desired attainment level of each objective function. Consider the FGP problem

Find X
to satisfy $G_i(X) \underset{\sim}{\geq}_i$, i = 1,2,...,m

$$\text{Subject to} \quad AX \leq b, \quad X \geq 0 \tag{2}$$

where X is an n-vector with components χ_1 , χ_2 ,$\cdots$, χ_n and $AX \leq b$ is a system of linear constraints in vector notation.

The symbol "$\underset{\sim}{\geq}$" refers to the fuzzification of the aspiration level. The i-th fuzzy goal $G_i(X) \underset{\sim}{\geq} g_i$ in problem (2) signifies that the decision maker is satisfied even if less than the goal g_i is achieved up to a certain tolerance limit . A linear membership function μ_i for the i-th fuzzy goal $G_i(X) \underset{\sim}{\geq} g_i$ can be expressed according to Zimmermann [13] as:

$$\mu_i = \begin{cases} 1 & \text{if} G_i(X) \geq g_i, \\ \dfrac{G_i(X)-L_i}{g_i-L_i} & \text{if} L_i \leq G_i(X) \leq g_i, \\ 0 & \text{if} G_i(X) \leq L_i, \end{cases} \tag{2.a}$$

Where L_i is the lower tolerance limit for the fuzzy goal $G_i(X)$. In the case of the goal $G_i(X) \lesssim g_i$ membership function is defined as:

$$\mu_i = \begin{cases} 1 & \text{if} G_i(X) \leq g_i, \\ \dfrac{U_i - G_i(X)}{U_i - g_i} & \text{if} g_i \leq G_i(X) \leq U_i, \\ 0 & \text{if} G_i(X) \geq U_i, \end{cases} \tag{2.b}$$

where U_i is the upper tolerance limit.

The additive model of the FGP problem (2) is formulated by adding the membership functions together as :

$$\text{Maximize} \quad V(\mu) = \sum_{i=1}^{m} \mu_i$$

Subject to:

$$\mu_i = \frac{G_i(X) - L_i}{g_i - L_i}$$
$$AX \le b_i$$
$$\mu_i \le 1,$$
$$X, \mu_i \ge 0, i = 1,2,\ldots,m \qquad (3)$$

where $V(\mu)$ is called the fuzzy achievement function or fuzzy decision function.

We now consider that there exist priority levels , i.e the goals are ranked according to the rule if $r < s$ then the goal $G_r(X)$ has higher priority than the goal $G_S(X)$ [4], the problem is subdivided into K subproblems, where K is the number of priority levels. In the first subproblem, only the fuzzy goals belonging to the first priority level are to be considered and solved using the simple additive model as mentioned above. But at other priority levels the membership values achieved earlier for higher priority levels are imposed as additional constraints. In general the i-th subproblem becomes

$$\text{Maximize } V(\mu) = \sum_s (\mu_s)_{p_i}$$

subject to

$$\mu_s = \frac{G_s - L_s}{g_s - L_s} \quad ,$$
$$AX \le b,$$
$$(\mu)\, p_r = (\mu^*)\, p_r \,, r = 1,2,\ldots, j-1,$$
$$\mu_s \le 1,$$
$$X, \mu_i \ge 0, i = 1,2,\ldots,m \qquad (4)$$

where $(\mu_s)p_i$ refers to the membership functions of the goals in the i-th priority level and $(\mu^*)p_i$ is the achieved membership value in the r-th ($r \le j\text{-}1$) priority level.

Problem Formulation

The development situation in Saudi Arabia is formulated as a detailed empirical application of FGP model in which three objectives are considered : reducing the size of foreign labor to a predetermined level, increasing the rate of growth of gross domestic product (GDP), and diversifying sources of national income.

Priority structure and definition of variables

P_1 : achieve desired level of foreign labor

P_2 : achieve desired level of gross domestic product (GDP).

P_3 : achieve desired level of diversification.

The endogenous variables are :

X_i = Total domestic production in the i^{th} economic sector[1] .

X_{ij} = The intersector flow of the i^{th} commodity to the j^{th} sector (i.e. intermediate demand).

M_i = The imports of the i^{th} sector (where $M_i = m_i X_i$).

The Exogenous variables are:

G_i = Government consumption expenditure in the ith sector

C_i = Private consumption expenditure in the ith sector

l_i = Fixed capital investment in the ith sector

E_i = Exports of the ith sector

I = Total fixed capital investment

F = The transfer payments from abroad

T = The transfer payment and investment abroad

Definition of constants:

a_{ij} = Current input requirements from the j^{th} sector per unit of output in the i^{th} sector (i.e. input-output coefficients where $a_{ij} = X_{ij}/X_j$).

m_i = Import-output coefficients.

k_j = Capital-output coefficients.

b_{sj} = Units of labor skill s required to produce one million Saudi Riyals (SR) worth of output by sector j.

c_j = Unit contribution of the jth sector to GDP.

The Fuzzy goal programming model

Maximize $(\mu_{Rs})\, p_1 + (\mu_{GDP})p_2 + (\mu_D)p_3$ (5a)

subject to $(\mu_{RS})\, P_1 = \dfrac{U_{Rs} - G_{RS}}{U_{RS} - R_S}$ (5b)

$$\mu_{GDP} = \frac{G_{GDP} - L_{GDP}}{GDP - L_{GDP}} \tag{5c}$$

$$\mu_D = \frac{U_D - G_D}{U_D - D} \tag{5d}$$

[1]The economy is divided into the following eleven sectors:
(1) Agriculture, (2) mining, (3) manufacturing, (4) utilities, (5) construction, (6) trade, (7) transport, (8) finance, (9) service, (10) government services, and (11) oil.

$$(1+m_i)X_i - \sum_{j=1}^{11} a_{ij}X_j \geq C_i + I_i + G_i + E_i \quad (5e)$$

$$\sum_{j=1}^{11} k_j X_j \leq I \quad (5f)$$

$$T + \sum_{j=1}^{11} m_j X_j \leq \sum_{i=1}^{11} E_i + F \quad (5g)$$

$$(\mu_{Rs})_{P1} = (\mu^*_{Rs})_{P1} \quad (5h)$$

$$(\mu_{GDP})_{P2} = (\mu^*_{GDP})_{P2} \quad (5i)$$

$$(\mu_D)_{P3} \leq 1 \quad (5j)$$

and

$$X_j \geq 0, 0 \leq \mu_{Rs}, \mu_{GDP}, \mu_D \leq 1; S = 1,2,3,4; i, j = 1,2,\ldots,11 \quad (5k)$$

where, $(\mu_{Rs})p_i$, $(\mu_{GDP})p_2$, and $(\mu_D)p_3$ refer to the membership functions of the labor skill s goal in the first priority level, the GDP goal in the second priority level, and the diversification goal in the third priority level respectively.

U_{Rs} is the upper tolerance limit of labor skill s.

G_{Rs} is the total number of labor skill s required to produce the total domestic production (i.e. $G_{Rs} = \sum_{j=1}^{11} b_{sj} X_j$).

R_s is the specified labor supply skill s.

G_{GDP} is sectors contribution to GDP, where $G_{GDP} = \sum_{j=1}^{11} c_j X_j$

L_{GDP} is the lower tolerance limit of gross domestic product.

GDP is the desired level of gross domestic product.

U_D is the upper tolerance limit of the diversification goal.

G_D is the difference between the contributions of the producing sectors (agriculture, mining, manufacturing) and that of oil (i.e. $G_D = \sum_{j=1}^{3} c_j X_j - c_{11} X_{11}$).

D is the desired level of diversification.

Equation (5e) refers to the supply-demand balance constraints which state that total sectoral supply (production and imports) must equal or exceed total sectoral demand (intermediate demand, private consumption expenditure, government consumption expenditure, fixed capital investment, and export).

Equation (5f) refers to the investment constraint which assumes that sectoral investment demand is a linear function of domestic production. Supply of investment is exogenously specified.

Equation (5g) is the balance of payment constraint which states that the sum of transfer payments and investment abroad, and the total amount of imports must be less than or equal to the sum of all sectoral exports and the transfer payment from abroad.

$(\mu_{R_S}{}^*)p_1$ is the achieved membership value in the first priority level.
$(\mu_{GDP}{}^*)p_2$ is the achieved membership value in the second priority level.

Results and Discussion

The model developed in the previous section is applied to real data obtained from official government and international sources as a case study and presented in the Appendix. The model is solved by standard simplex procedure as outlined by Tiwari *et al.* [10]. The sector domestic production in millions Saudi Riyals for 1984 are as follows:

Agriculture = 19,966.269 SR
Mining = 9892.301 SR
Utilities = 2004.306 SR
Construction = 55,636.633 SR
Transport = 32,673.220 SR
Service = 20,374.928 SR
Government service = 73731.174 SR
Manufacturing = 98,990.965 SR
Trade = 31,752.325 SR
Finance = 57327.482 SR
Oil = 132,469.618 SR

with achieved goals value:
Professional (G_{R1}) = 583,425.6 person
Office workers (G_{R2}) = 1,020,120 person
Skilled labor (G_{R3}) = 1,451,683 person
Unskilled labor (G_{R4}) = 1,410,006 person
GDP = 357581 SR Diversification(D) = 27827.85 SR

and memberships values:
$\mu_{R1} = .654$ $\mu_{R2} = 1.000$ $\mu_{R3} = 1.000$ $\mu_{R4} = 1.000$
$\mu_{GDP} = .685$ $\mu_D = .190$

These results represent the sectoral economic activities for the planning period of 1984. These activities represent the most efficient use of resources given the assumed development goals, priority patterns, labor and financial resources available. The results also indicate that the level of attainments of the established development goals of the

various labor skills, GDP, and diversification. The solution permits complete achievement of the office workers goal, the skilled labor goal, and the unskilled labor goal as indicated by corresponding membership values for these goals. However, the professional labor goal, the GDP goal, and the diversification goal have not been achieved fully as indicated by their corresponding membership values.

The professional labor goal was set at 563,900 person but the economic activities require about 583,425 persons which indicates a need for additional workers of about 19,525 persons. Similarly the GDP goal was underachieved by about 6.3%. The GDP goal was set at 381,592.3 million Saudi Riyal, but the achieved GDP level was 357,581.00 million. The underachievement of this goal is indicated by the GDP membership value of 0.685. The diversification goal refers to creating a viable modern economy that is less dependent on the oil sector. This goal involves pushing the contribution share of the producing sectors of agriculture, mining, and manufacturing in GDP to a level that is equal to 60% or higher than that of the oil sector.

The results indicate that this goal cannot be achieved fully under the limits imposed on the levels of the various labor skills and the specified GDP target. The combined GDP contributions of the producing sectors fall far below the GDP contribution of the oil sector such that the diversification goal is underachieved by about 40,523 million Saudi Riyal.

Conclusion

This paper presents a formulation and solution to the resource allocation problem in Saudi Arabia using fuzzy goal programming approach. Under this approach, three conflicting development goals under various sets of constraints on the economic system are considered.

The approach suggested in this paper can be used in the planning process of economic activities in developing countries. The practical application of the model will depend on the proper specification of the functional form of the economic system and will require detailed reliable statistical data. The model can be thought of as a supporting aid in developing and evaluating alternative strategies for allocating scarce economic resources.

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APPENDIX

Complete input data for fuzzy goal programming problem at final stage

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	μ_{R1}	μ_{R2}	μ_{R3}	μ_{R4}	μ_{GDP}	μ_D	T	RHS
Obj												1	1	1	1	1	1		xxxxxx
C1	3.922	.065	.5271	9.3298	2.0195	2.2212	1.777	.3015	5.2818	.8069	.0624	56390						=	620290
C2	7.095	.1183	.9536	16.8735	3.6537	4.0189	2.1307	.5456	9.5555	1.4599	.1129		102100					=	1112220
C3	10.097	.1688	1.357	24.013	5.1994	5.7189	3.0321	.7764	13.5981	2.0775	.1606			145297				=	1596980
C4	9.807	.1637	1.3181	23.3245	5.05	5.5545	2.9452	.7541	13.2075	2.0178	.1561				141104			=	1551110
C5	1.363163	-.052579	-.07294	-.029482	-.012367	-.004825	-.012506	-.024907	-.049598	-.001016	-.001614							>=	15440.657
C6	-.020788	1.022825	-.033023	-.013351	-.005599	-.002184	-.005662	-.011276	-.022455		-.000731							>=	4640.665
C7	-.311862	-.357115	1.22762	-.200253	-.083998	-.032769	-.08494	-.169166	-.336866	-.130955	-.010962							>=	75204.31
C8	-.002017	-.002306	-.0032	-.998705	-.000543	-.000212	-.000549	-.001093	-.002176	-.002462	-.000071							>=	1269.05
C9	-.004401	-.00504	-.006991	-.002824	.998815	-.000462	-.001199	-.002387	-.004754	-.054414	-.000155							>=	50415.18
C10	-.001418	-.001625	-.002253	-.000908	-.000382	.999851	-.000386	-.000769	-.001532	-.003466	-.00005							>=	31107.02
C11	-.021662	-.024806	-.034411	-.013911	-.005835	-.002276	1.06673	-.011751	-.023399	-.01644	-.000761							>=	27881.08
C12	-.086503	-.099054	-.137412	-.055549	-.023299	-.009089	-.02356	.953077	-.093439	-.038947	-.003041							>=	30683.69
C13	-.004476	-.005125	-.007109	-.002874	-.001205	-.00047	-.001219	-.002428	.9938	-.01005	-.00016							>=	18375.87
C14										1.07367								>=	79162.95
C15	-.02328	-.03113	-.03551	-.00926	-.00412	-.02565	-.01235	-.01163	-.03347	-.72805	.99913							>=	71569.43
C16	31879	03266	1.08317				.07664			1.862								<=	253704
C17	863	.210	1.536	110	4.954	3.348	2.744	2.554	.950		.310							<=	850001
C18												1						=	.654
C19													1					=	1
C20														1				=	1
C21															1			=	1
C22	4461	2549	.24461	.67928	.9771	.98722	79755	.5858	88821	.40944	.95828					-76318.46		=	305273.84
C23	-.4461	-.2549	-.24461								.574968						50000	=	50000
C24																1		=	.685
C25																	1	<=	1

منهج برمجة أهداف مشوشة لمشكلة توزيع موارد : حالة دراسية

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ملخص البحث. تقدم هذه الورقة نموذج برمجة أهداف مشوشة لتحليل وحل مشكلة توزيع الموارد الاقتصادية في المملكة العربية السعودية كحالة دراسية، لندرة تطبيقات نماذج برمجة الأهداف المشوشة في أدب تخطيط التنمية على مستوى الاقتصاد الكلي. تهدف هذه الورقة إلى وصف موجز لمشكلة توزيع الموارد بالمملكة العربية السعودية، وتقترح نموذج برمجة أهداف مشوشة. وقد تم تطبيق النموذج على بيانات حقيقية، حيث تشير النتائج إلى أن النموذج المقترح يقدم أداة ملائمة لمساندة عملية تطوير وتقييم الاستراتيجيات البديلة لتوزيع الموارد في الدول النامية.