نماذج النمو المفتولة: تطبيقاتها على القطاعات الاقتصادية في المملكة العربية السعودية

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ملخص البحث. تتمثل الطريقة المستخدمة في مقارنة معدّلات النمو لفترات مختلفة داخل سلسلة زمنية معينة، في تقدير معدّلات نمو أُسِّيَّة مستقلة لكل فترة. ولكن غالبًا ما تكون هذه النماذج غير متصلة، مما ينتج عن ذلك عدم الإنسجام بين تقديرات النمو للفترة كلها وتقديرات النمو للفترات المختلفة. تطبق هذه الدراسة النماذج الأُسِّيَّة المفتولة لتقدير معدلات النمو، خلال خطط التنمية الثلاثة الماضية، لمختلف قطاعات الاقتصاد بالمملكة العربية السعودية. وتبين النتائج إزالة عدم الانسجام بين تقديرات النمو، وبذلك تعتبر هذه الطريقة أفضل من غيرها لمقارنة معدّلات النمو مع سهولة تقديرها عن طريق برامج الانحدار الخطي المتعدد.

growth model estimates are better than these averages, since they are less sensitive to deviant observations.

Although, they have nice features, kinked growth models suffer from three shortcomings. First, despite that they diminish the impact of exceptional observations with respect to other procedures, or other models, they are still considered to be sensitive. Second, in case knot points are unknown, growth rate estimates may change drastically by changing knot locations, (see [7]). The situation studies here has known knot points, since development plans have known endpoints. As a third important limitation, kinked models are particularly ill-adapted to extrapolaiton beyond the observed range of the series. Because of this, they may be of limited usefulness in econometric models designed for forecasting.

For estimation and comparison of subperiod growth rates of time series, kinked growth constitute a simple, easy to fit, and a better estimation procedure. It is nice addition to the tool box of standard statistical methods with potential applications in different fields of research.

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Such inconsistencies are due to the discontinuity bias existing in model (4). This resulted in a decrease of subperiod growth rates for FIS and an increase of those for AGF. It happens that, during the first development plan AGF grew faster than FIS, giving an upward bias to the discontinuous subperiod growth rates. However, discontinuity at the first knot point initiated a downward bias in the first development plan for FIS sector. Similar arguments can be given for the comparison between NOG and TSC.

Kinked growth models remove these discordances, as it will be discussed in the following section.

Kinked Growth Model Estimates

Model (5) is fitted to the same five economic time series, with ln OAG adjustment. Table 3 gives growth rate estimates, from model (5), for the three development plans. We notice from Table 3 that the inconsistency in the NOG estimates has been removed, where subperiod growth rates are not all less than the whole period growth rate. also, inconsistencies in the comparison between AGF and FIS, and between NOG and TSC are removed.

Thus, after eliminating the discontinuity bias, kinked growth models provide a better basis for growth rate comparison of the three development plans, by removing discordances between growth rates in the whole period and its subperiods.

Table 3. Sectoral growth rate estimates (%), from the kinked model, for the three development plans.

	AGF	NOG	TSC	FIS	GDP
β_1	–11.25	31.90	8.58	11.57	4.76
	(0.044)	(0.052)	(0.025)	(0.065)	(0.011)
β_2	17.32	22.76	16.49	31.74	7.35
	(0.028)	(0.033)	(0.016)	(0.042)	(0.007)
β_3	15.33	–2.21	9.30	0.69	5.93
	(0.037)	(0.044)	(0.021)	(0.055)	(0.009)
R^2	0.95	0.98	0.99	0.95	0.99
F	47.11	122.5	231.1	55.04	1495.
DW	1.68	1.42	2.00	1.79	1.78

Concluding Remarks

It has been our purpose in this article to demonstrate the advantages of kinked growth models compared to conventional discontinuous models, by applying them to some economic sectors in the Kingdom of Saudi Arabia.

Achievements of different development plans, such as in [6], are discussed by means of simple yearly growth rate computations, and their averages. Kinked

Table 1. Sectoral growth rate estimates (%), from the growth model, for the whole period 1970-1985.

	AGF	NOG	TSC	FIS	GDP
β	14.15	13.87	12.21	17.31	6.42
	(0.057)*	(0.225)	(0.011)	(0.019)	(0.003)
AR(1)	0.744	0.871	0.393	0.321	–
	(0.175)	(0.214;	(0.316)	(0.176)	–
R^2	0.90	0.95	0.98	0.96	0.99
F	32.46	83.57	171.63	84.18	2545.
DW	1.60	1.62	1.59	1.52	1.65

R^2: R-squared, F: F-statistic, DW: Durbin-Watson statistic.
* Standard errors of β-coefficients are given in parentheses.

Then model (4) is fitted to the same five economic time series, with the adjustment of ln OAG. Table 2 gives the summary of growth rate estimates, using model (4), for the three development plans, with model related statistical measures. By simple examination of Tables 1 and 2, we notice that there are some discordances.

First, Table 1 indicates that the estimated whole period growth rate for NOG sector is 13.87, but in Table 2 all subperiod growth rates for NOG, (13.80, 8.96, and 4.13), are less than this estimate. Second, Table 1 indicates that FIS has experienced more rapid growth than AGF, 17.31% as opposed to 14.15%. However, Table 2 does not agree with these estimates, where we find that AGF sector experienced a better growth in the first and third subperiods, and a similar growth in the second. A similar discordance occurred between NOG and TSC sectors. The GDP sector is included here for completeness.

Table 2. Sectoral growth rate estimates (%), from the discontinuous model, for the three development plans.

	AGF	NOG	TSC	FIS	GDP
β_1	4.85	13.80	13.34	2.14	2.56
	(0.077)	(0.049)	(0.055)	(0.153)	(0.024)
β_2	25.80	8.96	19.98	25.82	7.19
	(0.038)	(0.024)	(0.027)	(0.074)	(0.019)
β_3	6.56	4.13	7.51	4.93	7.95
	(0.064)	(0.040)	(0.045)	(0.126)	(0.020)
R^2	0.97	0.99	0.99	0.96	0.99
F	51.55	460.9	163.7	33.50	934.5
DW	2.86	1.92	2.66	2.18	1.99

$$\ln y_t = \alpha_1 D_1 + \alpha_2 D_2 + \alpha_3 D_3 + \beta_1 D_1 t + \beta_2 D_2 t + \beta_3 D_3 t + e_t. \quad (4)$$

The estimated growth rates from model (4), β_1, β_2 and β_3 are the same estimators as those given by separate exponential growth models for each subperiod. In model (4), discontinuity is obviated by two linear restrictions:

$$\alpha_1 + \beta_1 k_1 = \alpha_2 + \beta_2 k_1$$

$$\alpha_2 + \beta_2 k_2 = \alpha_3 + \beta_3 k_2$$

Substituting for α_2 and α_3, we obtain the following kinked growth model for three subperiods:

$$\ln y_t = \alpha_1 + \beta_1(D_1 t + D_2 k_1 + D_3 k_1) + \beta_2(D_2 t - D_2 k_1 - D_3 k_1 + D_3 k_2) + \beta_3(D_3 t - D_3 k_2) + e_t \quad (5)$$

The OLS estimates of β_1, β_2 and β_3 provide adequate growth rate estimates for the first, second, and third subperiod, respectively.

The number of subperiods in a time series will vary between studies, and depends on the comparisons of interest. For the generalised kinked growth model, with m subperiods, the reader is referred to [4].

Economic Series and Growth Rates

Data were compiled from [5], covering the three previous development plans : 1970-1975, 1976-1980, and 1981-1985. Economic series (in Milions of S.R.) included in this study are:

AGF : Agriculture, forestry and fisheries,

NOG : Minerals other than crude oil and natural gas,

TSC : Transport, storage, and communications,

FIS : Financial servies,

GDP : Gross domestic poroduct,

OAG : Mining of crude oil and natural gas.

The cost of living index, with base year 1983, was used to deflate the series. Model (1) and (4) are applied to the first five economic time series. Since the development of the Saudi economy is driven by the oil and gas sector, OAG, ln OAG is included in both models as an independent variable to remove this sector's effect, and to estimate real growth rates. Table 1 gives growth rates estimated by model (1) for the whole period, 1970-1985, with related statistical measures. Note that in Table 1, model (1) is corrected for positive serial correlation, by the use of an autoregressive error process of order one, denoted by AR(1).

In this article, kinked growth models are presented, and are applied to Saudi Arabia economic sectors, where the subperiods are the three development plans: 1970-1975, 1976-1980, and 1981-1985. Results suggest that kinked growth models provide better growth rate estimates to compare the achievements of the three development plans.

Kinked Growth Models

The usual technique for estimating growth rates in time series is to fit an exponential growth model:

$$\ln y_t = \alpha + \beta t + e_t \; ; t = 1,\ldots,n \tag{1}$$

where $\ln y_t$ is the natural logarithm of y_t, β is the exponential growth for the whole period and e_t the error term, which is $N(0, \sigma^2)$. Model (1) reflects the situation where there are no subperiods.

A model with two subperiods is used when time series are broken in two segments at a knot point k. Dummy variables are used in model (1) to estimate growth rates in the two subperiods, such that we have the following model:

$$\ln y_t = \alpha_1 D_1 + \alpha_2 D_2 + \beta_1 D_1 t + \beta_2 D_2 t + e_t \tag{2}$$

where D_i is a dummy variable whose value is 1 in the i^{th} subperiod and 0 otherwise. Estimates β_1 and β_2 provide growth rate estimates for the first and second subperiod, respectively. Unfortunately model (2) will, in general, be discontinuous at knot point k. However, discontinuity can be obviated if the values of the right and left branches of the function at the knot point are equated, such that

$$\alpha_1 + \beta_1 k = \alpha_2 + \beta_2 k$$

Substituting for α_2 in model (2), and noting that $\alpha_1 D_1 + \alpha_1 D_2 = \alpha_1$, we get the kinked growth model with two subperiods,

$$\ln y_t = \alpha_1 + \beta_1 + \beta_1(D_1 t + D_2 k) + \beta_2(D_2 t - D_2 k) + e_t \tag{3}$$

Model (3) is a multiple regression model in which the dependent variable $\ln y_t$ is regressed on two composite variables whose values are constructed from the dummy variables, the knot point k, and trend t. The ordinary least squares (OLS) estimates of β_1 and β_2 give growth rates for the first and second period, respectively. Moreover, goodness of fit, significance tests and other related statistics for the kinked growth model are those obtained from multiple regression.

To compare growth rates of three subperiods, a situation of interest here for the three development plans, a kinked growth model with two knot points is used. The discontinuous model for this situation is

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Kinked Growth Models: An Application to Saudi Arabia Economic Sectors

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Abstract. For intertemporal growth rate comparisons, usually separate exponential trends lines are estimated for each subperiod of a time series. Unfortunately, these trend lines are likely to be discontinuous, which sometimes implies discordances between whole period and subperiod estimated growth rates. This article applies and adopts kinked exponential models, for subperiod growth rates estimation, to Saudi Arabia economic sectors. These models remove discordances, and are then a better basis for intertemporal growth rate comparisons. Besides, they are easily fitted by any standard package for multiple linear regression.

Introduction

Throughout its first two development plans, 1970 – 1980, the Kingdom of Saudi Arabia adopted a balanced strategy between the different sectors of the national economy. The third development plan, 1981 – 1985, is characterised by the emphasis on initiating changes in the economy, by directing the greater part of capital and manpower towards productive sectors such as agriculture, industry and minerals.

In order to compare the achievements of the three development plans, we need appropriate growth rate estimates. The usual procedure for estimating growth rates in the subperiods of a time series is to fit separate exponential trend lines to each subperiod within the time series. This procedure allows discontinuity between the trend lines, which can result in some discordances, such as subperiod growth rates which all exceed, or are less than, the estimated growth rate for the whole period. Also, when comparing growth rates in two time series, it happens that the whole period growth rate of one time series is higher than the other, but all subperiod rates of the latter exceed those of the former.

To obviate discontinuity in piecewise regression [1] suggested imposing linear restrictions at knot points. For an introduciton of piecewise regression see [2, pp. 346-350). In [3], the same procedure is used for spline functions. Also, [4] used similar restrictions in exponential growth models to yield the kinked growth models.